TRINITY ✠ COLLEGE

Year 12 Mathematics Applications
Test 3   2022

Section 1   Calculator Free
Networks

STUDENT'S NAME _____

DATE: Wednesday 11th May 2022   **TIME:** 40 minutes   **MARKS**: 40

INSTRUCTIONS:
Standard Items: Pens, pencils, drawing templates, eraser

Questions or parts of questions worth more than 2 marks require working to be shown to receive full marks.

_____

1.   (4 marks)

A connected planar network has 6 edges and 5 faces.

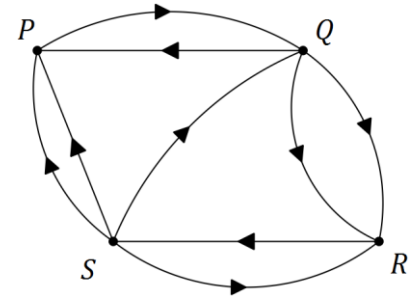(a)   How many vertices must it have?   [2]

(b)   Draw a planar graph with 6 edges and 5 faces.   [2]

2.   (6 marks)

   Calculate

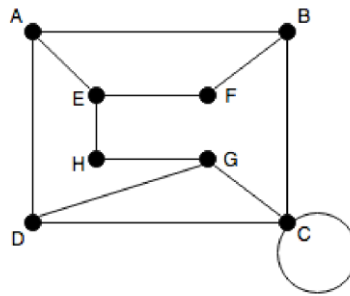   (a)   Construct the adjacency matrix for the following planar graph.   [2]

   (b)   Draw the graph for the following adjacency matrix.   [3]

	M	N	P	Q	R
M	0	0	0	1	1
N	0	0	1	1	0
P	0	1	0	1	0
Q	1	1	1	0	0
R	1	0	0	0	1

(c)   How many walks of length two are there from M to P?   [1]
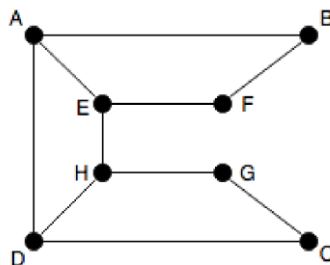
3.   (9 marks)

The diagram below shows a graph.

(a)   Determine, with reason, if this graph is a simple graph.   [2]

(b)   Determine, with reason, if this graph is traversable.   [2]

(c)   Identify a Hamiltonian cycle for the above graph, giving your answer as a sequence
   of vertices.   [1]

The graph has been altered to produce a subgraph.

(d)   Determine, with reason, if the edge EH has become a bridge.   [2]

(e)   Is the altered graph Eulerian, semi-Eulerian or neither? Justify you answer. [2]

4.   (5 marks)

The network below shows the connecting roads between three major towns, A, B and C. Jon lives in town A and his friend lives in town C. The weight represent distance, in kilometres between small towns and the three major towns A, B and C.
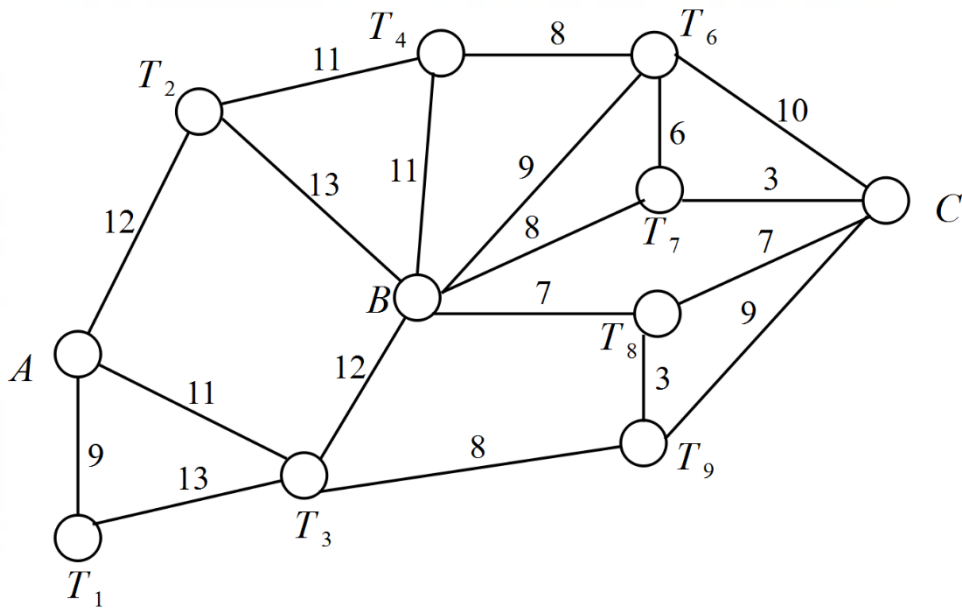

(a)   If Jon would like to visit his friend in town C, what is the shortest distance he must travel? Determine Jon's path to town C.   [3]

(b)   If Jon would like to pick up his friend Mark on his way in town B. What is the length of his short path now? Determine the path via town B.   [2]

5.   (6 marks)

Toby, a mathematics student at Trinity College, was employed as a guide for a cultural tour of Perth. He decided to use graph theory to plan the walking tour.

Below is a network he constructed in which the:
- vertices represent points of interest to be visited, and
- edges represent the most direct route between adjacent vertices.
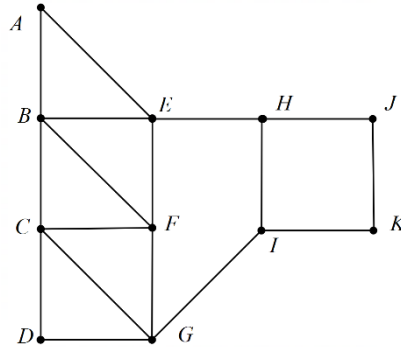

(a)   Use Euler's formula to verify the network if connected.   [1]

(b)   Toby, planned to take the group on a closed walk. Explain the meaning of a closed walk.   [1]

(c)   He also stated that the walk would qualify as a Hamiltonian cycle. State the two properties that makes the walk a Hamiltonian cycle.   [2]

(d)   Given that the walk started a Trinity College (G), mark the Hamiltonian cycle on the network below.   [2]
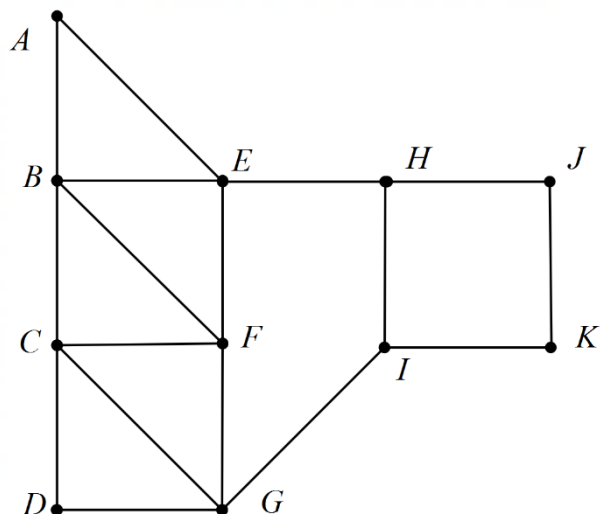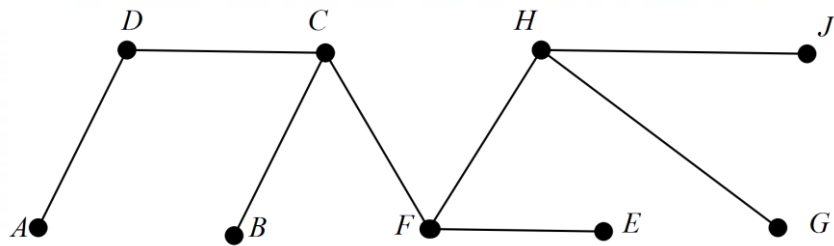
6.   (4 marks)

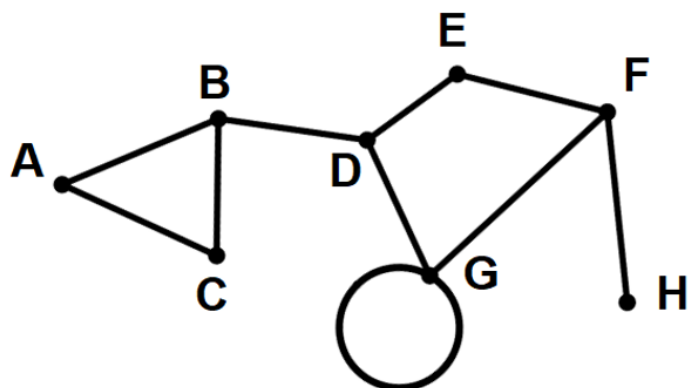(a)   Three friends, David, Eddy and Frank, like to play different card games.   [2]

- David enjoys playing Snap and Poker
- Eddy enjoys playing Snap, Poker and Black Jack
- Frank enjoys playing Poker and Rummy

Show this information as a bipartite graph

(b)   State whether the following graph is bipartite. If so, redraw the graph as a bipartite
graph.   [2]

7. (6 marks)

Answer the following questions using the graph above.

(a) Verify that the graph obeys Euler's formula. [2]

(b) List all the vertices that are of an odd degree. [2]

(c) State which edge(s) in the graph, if any are bridges. [2]